The Partnership Solution to the Common-Property Problem: An Experimental Test of the Theory

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Sharing the Catch Induces Free Riding

- Japanese fishermen (147 industries)
  - “The desire to avoid the various costs of crowding while operating in attractive fishing spots appears as the main reason stated by Japanese fishermen for adopting pooling arrangements” (Platteau and Seki, 2000)
- Hunter-Gatherers
  - “Work effort is extremely low in traditional societies and natural resources are not overexploited...whether traditional societies have introduced sharing consciously or chosen by accident, it appears to be a stable means to regulate resource use” (Kagi, 2001)

Socially Optimal Aggregate Fishing Effort

- The Goal
  - Maximize $\bar{p}f(X) - cX$
    or $\bar{p}XA(X) - cX$ where $A(X) = f(X)/X$.

    Normalize the fixed price to $\bar{p} = 1$. Socially optimal effort uniquely solves: $A(X^*) + X^*A'(X^*) - c = 0$
- The Problem
  - N independent agents will pursue their self interest.

Making the Pursuit of Self-Interest Within Partnerships Compatible with Achieving the Social Optimum

- If all N fishermen are grouped together into a single partnership where each person must pays his own costs but must share his catch equally with his partners, aggregate effort will be insufficient.
- If each of N fishermen is “grouped” into his own “solo” partnership with the same rules, aggregate effort will be excessive.
- As the number of groups increases from 1 to N, aggregate effort expands.
- Socially optimal effort can be achieved with an intermediate number (n^*) of groups.

The Motivations of Member k of Partnership i

- If partner k makes effort x_{ik} and his colleagues make effort Y_i^{-k} while aggregate effort is X, then the aggregate catch will be $f(X)$ and the portion going to partnership i will be $\frac{x_{ik}+Y_i^{-k}}{X}$.
- Since member k of partnership i must pay his own effort costs but gets an equal share of the catch of his partnership, individual k receives:

$$\pi_{ik} = \frac{1}{m_i}\left(\frac{x_{ik}+Y_i^{-k}}{X}\right)f(X) - cx_{ik}$$
$$\Leftrightarrow m_i\pi_{ik} = (x_{ik} + Y_i^{-k})A(x_{ik} + Y_i^{-k} + X_{-i}) - m_i cx_{ik}$$

The Best Response of Member *k* of Partnership *i*

- $A(X) + (x_{ik} + Y_i^{-k})A'(X) = m_i c$
- Compared to the hypothetical situation where he is not in a partnership but everyone else's efforts are unchanged, partner *k* reduces his effort for two reasons:
  1. Since under the partnership rules he *receives* a share of his partners' catch, the negative externalities that *k* imposes on colleagues now affect his own bottom line and this causes him to reduce effort.
  2. Since under the partnership rules he must *relinquish* to his colleagues all but $1/m_i$ of his additional catch but must pay the full cost of his additional effort, he has a further incentive to reduce effort.

Nash Equilibrium in the Effort Stage

We can rewrite the best responses as follows:

$A(X) + m_i \bar{x}_i A'(X) = m_i c$ for $i = 1, \ldots, n$ where $\bar{x}_i = \frac{x_{ik} + Y_i^{-k}}{m_i}$ and $X = \sum_{i=1}^{n} m_i \bar{x}_i$

- These n+1 equations may be solved for the n+1 variables (n mean effort levels and aggregate effort).
- Only the *mean effort* in each partnership is determined. That is, rearrangements of the same total effort within each partnership would also generate a Nash equilibrium.
- Henceforth, we assume that the effort of everyone within a given partnership is identical. Justification: the asymmetric equilibria are artifacts which would disappear if
  1. Any convexity in the cost function
  2. Any weight in the sharing rule of the partnership on relative effort.

Properties of Nash Equilibrium in the Effort Stage

If partnerships differ in size in the Nash equilibrium of a second-stage subgame, the larger the partnership,

1. the smaller the mean effort—Proof: $\frac{1}{m_i}A(X) + \bar{x}_i A'(X) = c$.
2. the smaller the payoff to each member—Proof: $\pi_{ik} = \frac{1}{m_i}[m_i \bar{x}_i A(X)] - c\bar{x}_i = \bar{x}_i(A(X) - c)$.

Summing the n first-order conditions to obtain $nA(X) - XA'(X) - cN = 0$, we can conclude that

- Aggregate effort depends only on the number of partnerships and not on the distribution of the N agents among these partnerships.
- Aggregate effort strictly increases with the number of partnerships.

Number of Partnerships Needed to Achieve the Social Optimum

Let $n^* = 1 + \frac{c(N-1)}{A(X^*)}$. Then, if $n = n^*$, aggregate effort in the partnership solution is socially optimal. Proof: Substitute into the equation defining aggregate effort with n partnerships,$nA(X) + XA'(X) - cN = 0$, to obtain $A(X) + XA'(X) - c = 0$. But the aggregate effort solving this equation is by definition X^*, the socially optimal effort.

- Note the integer problem: what if $n = 2.1$?
- When $c = 0$, partnership of the whole ($n^* = 1$) is socially optimal.
- As c increases, n* increases.

Why Everyone Takes Full Account of the Social Effects of His Actions in the Partnership Solution

If any player in any partnership increases his effort one unit, he increases social cost by c. Consider someone in partnership i. His action strictly *increases* the benefit to each of his $m_i - 1$ partners while it strictly *decreases* the benefit to each of the $N - m_i$ non-partners. Net effect?

- His action strictly increases the benefit to all his partners by $(1 - \frac{1}{m_i})(A(X) + X_i A'(X)) = A(X) + X_i A'(X)) - [\frac{1}{m_i}(A(X) + X_i A'(X))] = A(X) + X_i A'(X)) - c$.
- The action strictly reduces the benefit to outsiders by $(X - X_i)A'(X)$.
- Net effect: $A(X) + XA'(X) - c = 0$ for $X = X^*$. In that case, the agent internalizes all the net social costs and benefits of increasing his effort.

Stable Partnerships

Assume partners pick a partnership simultaneously.

- Partnership sizes must differ by at most 1 member (otherwise there would be a temptation to move to a smaller partnership).
  - This arrangement can always be achieved (for any specified number of partnerships $n = 1, \ldots N$).
- Solo production must always be inferior to team production (by two or more agents) or there would always be a temptation to go solo.
  - Assume 1 man-hour of team effort equals $1/\beta$ man hours of solo effort ($\beta \in [0, 1)$).
  - In some cases (e.g. whaling), a team is essential for production ($\beta = 0$.)
  - In other cases, β must be "*sufficiently small*" for there to be no temptation to go solo.
  - If β is larger, partnerships may still achieve a big improvement (but not the social optimum).

The Possibility of Multiple Equilibria at the First Stage

There may be multiple equilibria at the first stage. For example, if team production is essential ($\beta = 0$), there will be an equilibrium associated with any number of partnerships ($n = 1, \ldots, N$).

Suppose $N = 5$. There will be 5 equilibria:

- $n = 1 : \ \{5\}$
- $n = 2 : \ \{3, 2\}$
- $n = 3 : \ \{2, 2, 1\}$
- $n = 4 : \ \{2, 1, 1, 1\}$
- $n = 5 : \ \{1, 1, 1, 1, 1\}$

Implementing the Equilibrium in the First Stage: Voting

- Suppose the N agents vote on how many partnerships to establish— with the understanding that agents will be randomly dealt out to partnerships so that partnerships do not differ in size by more than one member.
- Since every participant strictly prefers that n^* partnerships form, n^* will be chosen in any reasonable voting game. Two examples:
  1. Every participant votes and one vote, chosen at random, dictates the collective choice.
  2. A sequential elimination tournament between pairs of choices with the agenda common knowledge at the outset.

Laboratory Tests

- 6 subjects in groups of equal size (groups of 6 or 3 or 2 or everyone solo).
- Subject must invest an endowment of 6 tokens across two "investment projects"
- An individual gets a fixed return of c for each token invested in Project A. His payoff from Project B is decreasing in the aggregate investment (X) in that project, increasing in his partnership's investment (Y_{-i}) and decreasing in the size (m_i) of the partnership.
- Payoff of subject k in partnership i: $\pi_{ik} = \frac{1}{m_i}(x_{ik} + Y_i^{-k})A(x_{ik} + Y_i^{-k} + X_{-i}) + (6 - x_{ik})c$
- Parameters: $N = 6,\ A(X) = 200 - 5X,\ c = 1, 20, 55, 100.$

Example

Let's say you invested 2 tokens in Project B and the other participant in your group invested 6 tokens in that project. In addition, let's say the total investment in Project B by the others that are not in your group is 14.

Then the total investment in Project B is 22 tokens. The return from Project B is (200 - 5.0*22) = 90, and the total you earn from Project B is (1/2)*90*8 = 360.

Since you invested 2 of your 6 tokens in Project B, the other 4 are automatically invested for you in Project A. Since the return there is 20 times the amount invested, you earn 80 tokens from Project A.

Your final earnings this round is the sum of your earnings from Project A and Project B. In the example, you would have earned 360 + 40= 440 tokens.

Please click "Continue" when you are ready.

Continue

Time Left [sec]: 56

Situation Analyzer: Groups of 2

If the Total Investment By Others in Your Group in Project B is

If the Total Investment By Others Outside Your Group in Project B is

Analyze This Situation

Your Choice	0	1	2	3	4	5	6
Your Earnings (A + B)	575.0	590.0	567.5	540.0	507.5	470.0	427.5

Return to Project B per token invested = 200 - 5.0 * (total number of tokens invested in Project B by all participants)

Your Earnings from Project B = (1/2) * Return to Project B per token invested * (total number of tokens invested in Project B by your group)

Return to Project A = 20

Your earnings from Project A = 20 * your investment in Project A

Decision Entry

Number of tokens: 6

Your investment in Project B:

Your best estimate of the investment of others in your group in Project B:

Your best estimate of the investment of others outside your group in Project B:

OK

Experimental Design

- 25 sessions in March 2009 at ISR Lab at UM
- Each session has 6 participants and a single cost
- In each session subjects play the game for 5 periods in each of four group sizes (periods 1-20), and then choose the group size by voting
- They vote 5 times for the group size. After each vote, they play once (periods 21-25)
- Equal sized groups: 1, 2, 3 or 6 subjects per group
- Random matching of subjects in the first 4 treatments of the session
- 5 of the 25 periods are selected for payments: 1 period of first 5 at random; 1 period of the next 5…
- Approximately $25 on average (plus $5 show up fee). $1=100 tokens

Experimental Design (cont’d)

- Foregoing is then repeated 5 times in different sessions with new sets of 6 subjects (30 subjects per cost treatment)
- In each repetition, order of the first 4 treatments is changed.

- All of the above is then repeated five times: 4 different costs (1,20,55,100) +1 extra treatment where cost of 20 is run again with a “dictatorial” voting procedure
- 150 participants in total (30 people per cost treatment *5 cost treatments)
- Both within and between subjects comparisons

Theoretical Predictions for cost = 1

Group size	Contributions
1	5.69
2	4.95
3	4.38
6	3.23
Socially efficient group size = 6 Socially efficient contribution= 3.32	

Mean Contributions cost = 1

Group size	Predicted	Actual
1	5.69	5.02
2	4.95	4.47
3	4.38	3.98
6	3.23	3.13

Average payoff in the first 4 treatments and the percentage of votes for each group size in the 5th treatment

cost = 1

Group size	Predicted payoff	Average payoff	Efficiency	Percentage of Votes
1	167.6	241.1	0.72	8
2	256	285.9	0.85	16
3	302.2	314.1	0.93	16.7
6	335.8	323	0.96	59.3
Socially efficient payoff = 336 Efficiency = (actual average payoff/max. possible) *100				

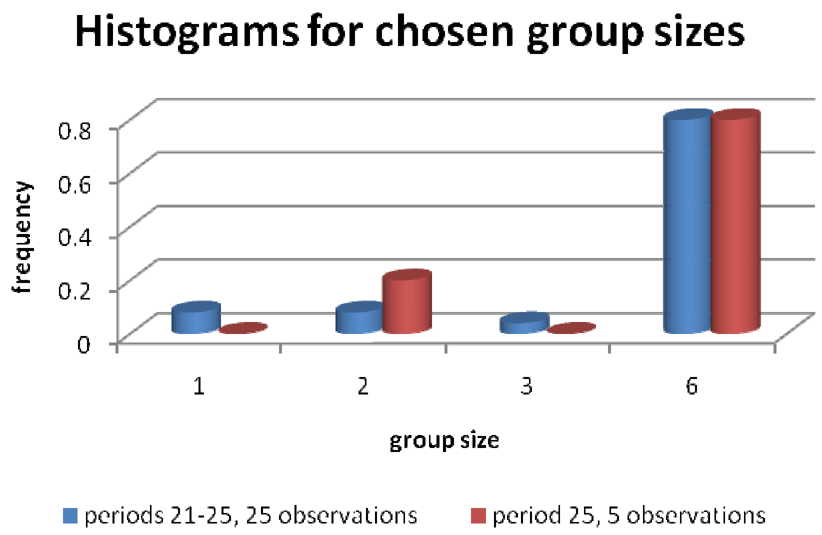
Efficiency in Group Selection
cost = 1

Actual average payoff	304.9
Maximum possible payoff	336
Efficiency (in %)	91
Efficiency = (actual average payoff/max. possible) *100	

Group selection, cost = 1
conditional on the chosen group size

Group size	Frequency (out of 25)	Average contribution	Predicted contribution	Average payoff	Predicted payoff
1	2	5.75	5.69	158.17	167.6
2	2	4.67	4.95	278	256
3	1	4.5	4.38	294	302.2
6	20	3.26	3.23	322.78	335.8
Socially efficient group size = 6 Socially efficient payoff = 336					

Group selection, cost = 1
all periods versus last period

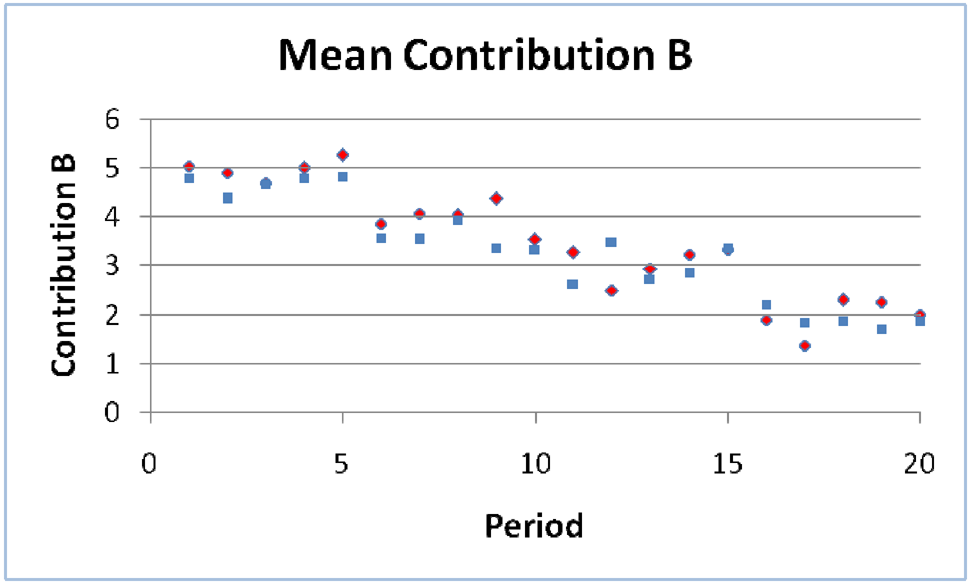


Theoretical Predictions for cost = 20

Group size	Contributions
1	5.14
2	4
3	3.11
6	1.33
Socially efficient group size = 3 Socially efficient contribution= 3	

Cost=20 (voting versus dictator)

voting = blue, dictator = red



- By using two-sample Wilcoxon rank-sum (Mann-Whitney) test, we cannot reject the hypothesis that the data from voting and dictator treatments are from populations with the same distribution for group sizes 3 and 6 (p-values are 1, and 0.21).

- For group sizes 1 and 2, contributions are significantly higher in the dictator experiment (p-values are 0.06, and 0.03).

Mean Contributions cost = 20

Group size	Predicted	Actual (Voting)	Actual (Dictator)
1	5.14	4.69	4.98
2	4	3.55	3.97
3	3.11	3.02	3.05
6	1.33	1.89	1.97

Average payoff in the first 4 treatments and the percentage of votes for each group size in the 5th treatment

cost = 20 (voting)

Group size	Predicted payoff	Average payoff	Efficiency	Percentage of Votes
1	252.2	294.9	0.76	12.7
2	360	369.6	0.95	39.3
3	389.6	376.5	0.97	39.3
6	306.7	340.7	0.87	8.7
Socially efficient payoff = 390 Efficiency = (actual average payoff/max. possible) *100				

Average payoff in the first 4 treatments and the percentage of votes for each group size in the 5th treatment
cost = 20 (dictator)

Group size	Predicted payoff	Average payoff	Efficiency	Percentage of Votes
1	252.2	268.1	0.69	9.3
2	360	353.6	0.91	18.7
3	389.6	371.7	0.95	40.7
6	306.7	346.7	0.89	31.3
Socially efficient payoff = 390 Efficiency = (actual average payoff/max. possible) *100				

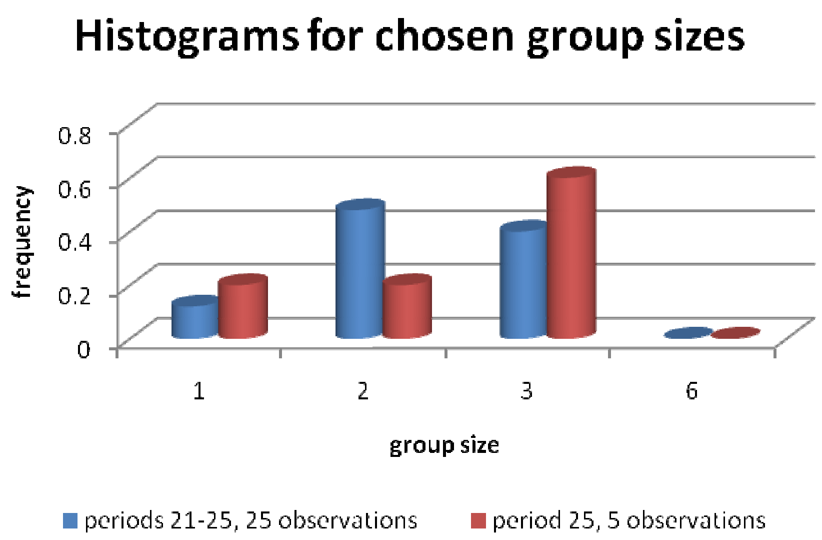
Efficiency in Group Selection (in 5th treatment) cost=20

	Voting	Dictator
Actual average payoff	362	362.2
Maximum possible payoff	390	390
Efficiency (in %)	93	93
Efficiency = (actual average payoff/max. possible) *100		

Group selection, cost = 20, voting conditional on the chosen group size

Group size	Frequency (out of 25)	Average contribution	Predicted contribution	Average payoff	Predicted payoff
1	3	4.89	5.14	266.11	252.2
2	12	3.74	4	368.54	360
3	10	3	3.11	382.83	389.6
6	0	-	1.33	-	306.7
Socially efficient group size = 3 Socially efficient payoff = 390					

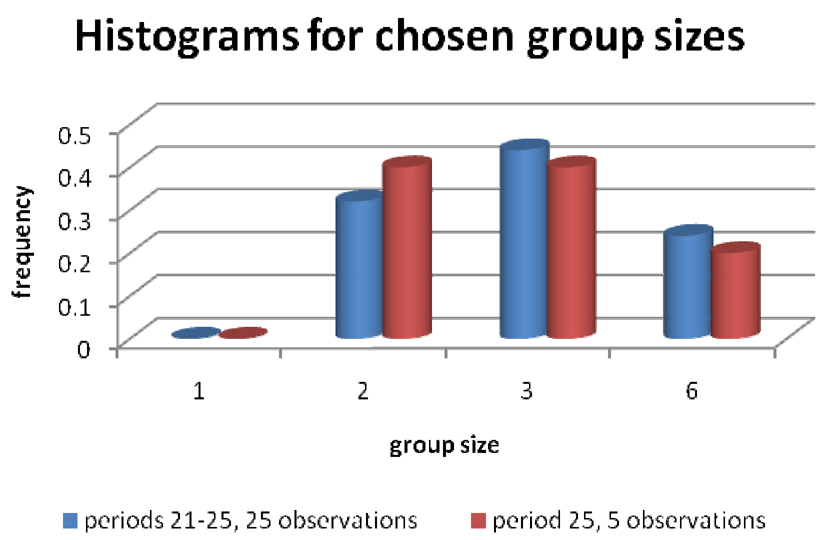
Group selection, cost = 20, majority voting
all periods versus last period



Group selection, cost = 20, dictator conditional on the chosen group size

Group size	Frequency (out of 25)	Average contribution	Predicted contribution	Average payoff	Predicted payoff
1	0	-	5.14	-	252.2
2	8	3.99	4	358.02	360
3	11	2.91	3.11	370.45	389.6
6	6	1.97	1.33	352.64	306.7
Socially efficient group size = 3 Socially efficient payoff = 390					

Group selection, cost = 20, dictator
all periods versus last period



Theoretical Predictions for cost = 55

Group size	Contributions
1	4.14
2	2.25
3	0.78
6	0
Socially efficient group size = 2 Socially efficient contribution= 2.42	

Mean Contributions
cost = 55

Group size	Predicted	Actual
1	4.14	3.89
2	2.25	2.34
3	0.78	1.42
6	0	1.15

Average payoff in the first 4 treatment and the percentage of votes for each group size

cost = 55

Group size	Predicted payoff	Average payoff	Efficiency	Percentage of Votes
1	415.8	426.6	0.84	7.3
2	504.4	497.2	0.98	57.3
3	424.6	464.5	0.92	11.3
6	330	443	0.88	24
Socially efficient payoff = 505.2 Efficiency = (actual average payoff/max. possible) *100				

Efficiency in Group Selection cost = 55

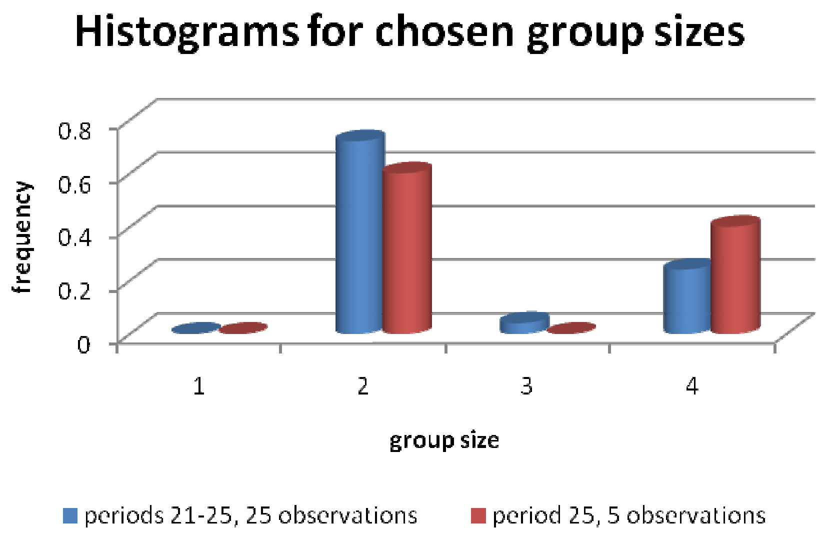
Actual average payoff	482
Maximum possible payoff	505.2
Efficiency (in %)	95
Efficiency = (actual average payoff/max. possible) *100	

Group selection, cost = 55
conditional on the chosen group size

Group size	Frequency (out of 25)	Average contribution	Predicted contribution	Average payoff	Predicted payoff
1	0	-	4.14	-	415.8
2	18	2.2	2.25	498.2	504.4
3	1	0.83	0.78	430	424.6
6	6	1.11	0	442.2	330
Socially efficient group size = 2 Socially efficient payoff = 505.2					

Group selection, cost = 55

all periods versus last period



Theoretical Predictions for cost = 100

Group size	Contributions
1	2.86
2	0
3	0
6	0
Socially efficient group size = 1 Socially efficient contribution= 1.67	

Mean Contributions
cost = 100

Group size	Predicted	Actual
1	2.86	2.93
2	0	1.26
3	0	1.17
6	0	0.81

Average payoff in the first 4 treatments and the percentage of votes for each group size in the 5th treatment

cost = 100

Group size	Predicted payoff	Average payoff	Efficiency	Percentage of Votes
1	640.8	624.6	0.92	22
2	600	670.4	0.98	24.7
3	600	670.9	0.98	33.3
6	600	656	0.96	20
Socially efficient payoff = 683.3 Efficiency = (actual average payoff/max. possible) *100				

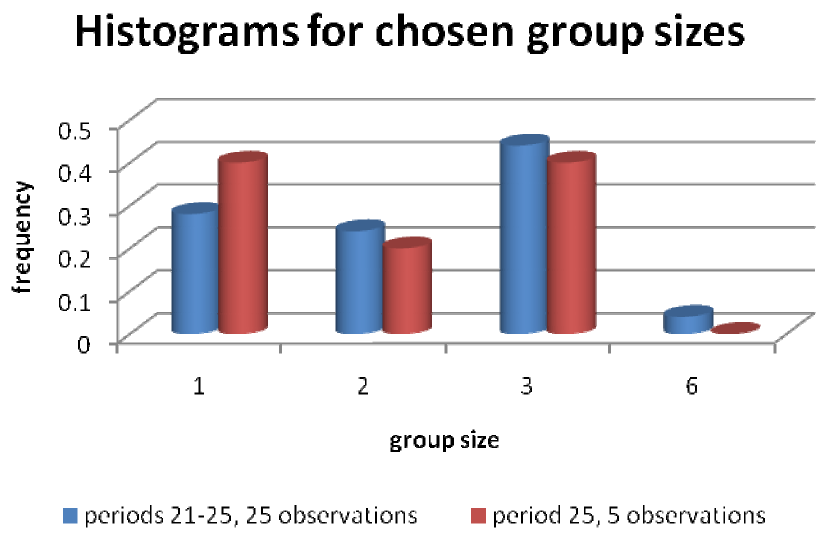
Efficiency in Group Selection cost = 100

Actual average payoff	655.9
Maximum possible payoff	683.3
Efficiency (in %)	96
Efficiency = (actual average payoff/max. possible) *100	

Group selection, cost = 100
conditional on the chosen group size

Group size	Frequency (out of 25)	Average contribution	Predicted contribution	Average payoff	Predicted payoff
1	7	2.67	2.86	648.1	640.8
2	6	1	0	658.61	600
3	11	0.85	0	660.6	600
6	1	0.5	0	642.5	600
Socially efficient group size = 1 Socially efficient payoff = 683.3					

Group selection, cost = 100
all periods versus last period



Future Work

- As c increases underinvestment in project B diminishes and becomes overinvestment in project B (actual compared to Nash). Why?
- Understand investment behavior when c=100. In that case, investing more in project A would raise a person's monetary payoff no matter what the conjecture about the other players' behaviors---suggesting his/her payoff is not entirely monetary.
- Does aggregate investment depend only on the number of groups---e.g. same investment with (5,1), (4,2), or (3,3)?
- Would subjects migrate from a large to a small group if given the opportunity?
- Would a subject set up a "solo" partnership if given the opportunity?